

Applying Statistics 5.3 Finding Probabilities

The distribution of scores on a biology final exam used in Applying Statistics 5.2 had a mean of 72 and a standard deviation of 8. What is the probability that a student selected at random will have a score less than 61? More than 80? Less than 98? To answer these questions, we must first calculate Z scores and then consult Appendix A. We are looking for probabilities, so we will leave the areas in proportion form.

The Z score for a score of 61 is

$$Z = \frac{X_i - \bar{X}}{s} = \frac{61 - 72}{8} = -\frac{11}{8} = -1.38$$

This score is a negative value (below, or to the left of, the mean), and we are looking for the area below. Using Table 5.4 as a guide, we will find the area below a negative score in column c. This area is 0.0838. Rounding off, we can say that the odds of selecting a student with a score less than 61 are only 8 out of 100. This low value tells us this would be an unlikely event.

The Z score for the score of 80 is

$$Z = \frac{X_i - \bar{X}}{s} = \frac{80 - 72}{8} = \frac{8}{8} = +1.00$$

The Z score is positive, and to find the area above (greater than) 80, we look in column c (see Table 5.4). This value is 0.1587. The odds of selecting a student with a score greater than 80 is roughly 16 out of 100, about twice as likely as selecting a student with a score of less than 61.

The Z score for the score of 98 is

$$Z = \frac{X_i - \bar{X}}{s} = \frac{98 - 72}{8} = \frac{26}{8} = +3.25$$

To find the area below a positive Z score, we add the area between the score and the mean (column b) to 0.5000 (see Table 5.4). This value is 0.4994 + 0.5000, or 0.9994. It is extremely likely that a randomly selected student will have a score less than 98. Remember that scores more than ± 3 standard deviations from the mean are very rare.

STATISTICS
IN EVERYDAY LIFE

Probability Theory in Action

If you enjoy games of chance—bingo, card games, the state lottery, or board games like Parcheesi—you can use your knowledge of the laws of probability to improve your performance and your likelihood of winning. This point was illustrated in the best-seller *Bringing Down the House*, later made into a movie called *21*. The book describes the adventures of a group of Massachusetts Institute of Technology students who used their understanding of probability to develop a system that almost guaranteed they could beat the house in the game of blackjack. They won hundreds of thousands of dollars before they were finally stopped by casino security.

SUMMARY

1. The normal curve, in combination with the mean and standard deviation, can be used to construct precise descriptive statements about empirical distributions that are normally distributed. This chapter also lays some important groundwork for Part II.
2. To work with the theoretical normal curve, we must transform raw scores into their equivalent Z scores. Z scores allow us to find areas under the theoretical normal curve (Appendix A).
3. We considered three uses of the theoretical normal curve: finding total areas above and below a score, finding areas between two scores, and expressing these areas as probabilities. This last use of the normal curve is especially germane because inferential statistics are centrally concerned with estimating probabilities in a fashion very similar to the process introduced in this chapter.

SUMMARY OF FORMULAS

FORMULA 5.1

Z Scores:

$$Z = \frac{X_i - \bar{X}}{s}$$

GLOSSARY

Normal curve. A theoretical distribution of scores that is symmetrical, unimodal, and bell-shaped. The standard normal curve always has a mean of 0 and a standard deviation of 1.

Normal curve table. Appendix A; a detailed description of the area between a Z score and the mean of any standardized normal distribution.

Z scores. Standard scores; the way scores are expressed after they have been standardized to the theoretical normal curve.

PROBLEMS

- 5.1 Scores on a quiz were normally distributed and had a mean of 10 and a standard deviation of 3. For each of the following scores, find the Z score and the percentage of area above and below the score.

X_i	Z Score	% Area Above	% Area Below
5			
6			
7			
8			
9			
11			
12			
14			
15			
16			
18			

- 5.2 Assume that the distribution of scores on a college entrance exam is normal, with a mean of 500 and a standard deviation of 100. For each of the following scores, find the equivalent Z score, the percentage of the area above the score, and the percentage of the area below the score.

X_i	Z Score	% Area Above	% Area Below
650			
400			
375			
586			
437			
526			
621			
498			
517			
398			

- 5.3 The senior class has been given a comprehensive examination to assess educational experience. The mean score on the test was 74 and the standard deviation was 10. What percentage of the students had scores

- a. between 75 and 85? _____
- b. between 80 and 85? _____
- c. above 80? _____
- d. above 83? _____
- e. between 80 and 70? _____
- f. between 75 and 70? _____
- g. below 75? _____
- h. below 77? _____
- i. below 80? _____
- j. below 85? _____

- 5.4 For a normal distribution where the mean is 50 and the standard deviation is 10, what percentage of the area is

- a. between the scores of 40 and 47? _____
- b. above a score of 47? _____
- c. below a score of 53? _____
- d. between the scores of 35 and 65? _____
- e. above a score of 72? _____
- f. below a score of 31 and above a score of 69? _____
- g. between the scores of 55 and 62? _____
- h. between the scores of 32 and 47? _____

- 5.5 At St. Algebra College, the 200 freshmen enrolled in introductory biology took a final exam on which their mean score was 72 and their standard deviation was 6. The following table presents the grades of 10 students. Convert each into a Z score and determine the number of people who scored higher or lower than each of the 10 students. (HINT: Multiply the appropriate proportion by N and round the result.)

X_i	Z Score	Number of Students Above	Number of Students Below
60			
57			
55			
67			
70			
72			
78			
82			
90			
95			

- 5.6 If a distribution of test scores is normal, with a mean of 78 and a standard deviation of 11, what percentage of the area lies

- below 60? _____
- below 70? _____
- below 80? _____
- below 90? _____
- between 60 and 65? _____
- between 65 and 79? _____
- between 70 and 95? _____
- between 80 and 90? _____
- above 99? _____
- above 89? _____
- above 75? _____
- above 65? _____

- 5.7 **[SOC]** A scale measuring prejudice has been administered to a large sample of respondents. The distribution of scores is approximately normal, with a mean of 31 and a standard deviation of 5. What percentage of the sample had scores

- below 20? _____
- below 40? _____
- between 30 and 40? _____
- between 35 and 45? _____
- above 25? _____
- above 35? _____

- 5.8 **[SOC]** On the scale mentioned in problem 5.7, if a score of 40 or more is considered "highly prejudiced," what is the probability that a person selected at random will have a score in that range?

- 5.9 For a math test on which the mean score was 59 and the standard deviation was 4, what is the probability

that a student randomly selected from this class will have a score

- between 55 and 65? _____
- between 60 and 65? _____
- above 65? _____
- between 60 and 50? _____
- between 55 and 50? _____
- below 55? _____

- 5.10 **[SOC]** The average number of dropouts for a school district has been 305 per year with a standard deviation of 50. What is the probability that the number of dropouts next year will be

- less than 250? _____
- less than 300? _____
- more than 350? _____
- more than 400? _____
- between 250 and 350? _____
- between 300 and 350? _____
- between 350 and 375? _____

- 5.11 **[CJ]** The local police force gives all applicants an entrance exam and accepts only those applicants who score in the top 15% on this test. If the mean score this year is 87 and the standard deviation is 8, would an individual with a score of 110 be accepted?

- 5.12 To be accepted into an honor society, a student must have a grade-point average (GPA) in the top 10% of the school. If the mean GPA at the school is 2.78 and the standard deviation is 0.33, which of the following GPAs would qualify?

3.20, 3.21, 3.25, 3.30, 3.35

- 5.13 In a distribution of scores with a mean of 35 and a standard deviation of 4, which event is more likely: that a randomly selected score will be between 29 and 31 or that a randomly selected score will be between 40 and 42?

- 5.14 After taking the state merit examinations for the position of school counselor and social worker, you receive the following information on the tests and on your performance. On which test did you do better?

School Counselor	Social Worker
$\bar{X} = 118$	$\bar{X} = 27$
$s = 17$	$s = 3$
Your score = 127	Your score = 29

- 5.15 A large university administers entrance exams to all entering students. The test is administered again as an exit exam for all graduates. The test results for one group of students are given in the table.

Freshman Year	Senior Year
$\bar{X} = 53$	$\bar{X} = 92$
$s = 7$	$s = 4$

- a. The scores for five students on both tests are listed below. For each student, calculate Z scores and determine whether the student performed better as a freshman or as a senior.

Student	Score, Freshman Year	Score, Senior Year
A	57	97
B	51	94
C	45	82
D	73	101
E	62	98

- b. Determine the probabilities that randomly selected students will have scores in each of these ranges:

Freshman Year	
Score	Probability
Less than 52	
Less than 57	
Between 40 and 50	
More than 51	
More than 62	

Senior Year	
Score	Probability
Less than 88	
Less than 98	
Between 80 and 100	
More than 97	
More than 85	